Research Guru
Online Journal of Multidisciplinary Subjects (ISSN : 2349-266X)
www.researchguru.net
Volume-16, Issue-2, September-2022

Bianchi type-II String Cosmological Models with Time Dependent Bulk Viscosity in General Relativity

Dinkar Singh Chauhan
Department of Mathematics, Mahadev Post-Graduate College, Bariyashanpur, Varanasi-221112, (UP) India
E-mail: dschauhan9616@gmail.com

R.S. Singh
Department of Mathematics, Post-Graduate College, Ghazipur-233001, (UP)India
E-mail: rss1968@rediffmail.com

Abstract-*Bianchi type-II string cosmological models with bulk viscous fluid for massive string are investigated. To obtain the determinate model of the universe, we assume that (i) $\rho + \lambda = 0$, where ρ is the energy density and λ is the string tension density (ii) relation between metric potentials $A = aB^n$. To obtain a more general solution, it is also assumed that the coefficient of bulk viscosity (ξ) is a power function of the expansion scalar (θ) i.e. $\xi = k\theta^n$, where k and m are positive constants. It is found that the power index m has significant influence on the string model. There is a big-bang start in the model when $m \leq 1$ but there is no big-bang start when $m > 1$. Behaviour of the models in the presence and absence of bulk viscosity are discussed. The physical implications of the models are also discussed in detail.*

Keywords: *Bianchi-II model, Massive string, Viscous fluid, Exact solution, Isotropy, Cosmology*

1. INTRODUCTION

Cosmic strings play an important role in study of the early universe. These strings arise during the phase transition after the big-bang explosion as the temperature goes down some critical temperature as predicted by grand unified theories (GUT) (Kibble 1976, 1980; Zel'dovich 1975, 1980; Everett 1981; Vilenkin 1981, 1985). It is believed that cosmic strings give rise to density perturbations which lead to formation of galaxies (Zel'dovich 1980). These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings. The general relativistic treatment of strings has been initially given by Letelier (1979, 1983) and Stachel (1980). Letelier (1983) defined the massive strings as the geometric strings (massless) with particles attached along its extension. The exact solutions of string cosmology for Bianchi type II, VI0, VIII and IX space-times have been studied by Krori et al. (1990) and Wang (2003). Patel et al. (1996) have investigated the integrability of cosmic string in context of Bianchi type-II, VIII and IX space-times. Roy and Banerjee (1995) dealt with LRS cosmological models of Bianchi type-II representing clouds of geometrical as well as massive strings. Yadav et. al. (2012) have studied Bianchi type-II massive string cosmological models with time-decaying Λ term. Sharma and Singh (2014) have investigated Bianchi type-II string cosmological model with magnetic field in f (R, T) gravity. Bianchi type-II oscillating string cosmological model in f (R, T) gravity is investigated by Rao and Prasanthi (2017). Recently, Mishra et al.

(2019) has studied bulk viscous string cosmological model in Saez-Ballester theory of gravitation.

In most of the cosmological models matter distribution in the universe is satisfactorily described by a perfect fluid due to large-scale distribution of galaxies in our universe. However, a realistic treatment of the problem requires the consideration of material distribution other than the perfect fluid. Cosmological models with bulk viscosity are important since bulk viscosity has a greater role in getting accelerated expansion of the universe popularly known as inflationary phase. It is well known that at an early stage of universe when neutrino decoupling occurred, the matter behaved like a viscous fluid (Kolb and Turner 1990). Weinberg (1972) derived general formulae for bulk and shear viscosity and used these to evaluate the rate of cosmological entropy production. He deduce that the most general form of the energy-momentum tensor, allowed by rotational and space inversion invariance, contains a bulk viscosity term proportional to the volume expansion of the model. Padmanabham et. al. (1987) also noted that viscosity may be relevant for the future evolution of the universe. Recently, Avchar et al. (2021) has studies axially symmetric bulk viscous cosmological model in f (R, T) theory of gravity.

Bianchi type-II space-time has a fundamental role in constructing cosmological models suitable for describing the early stages of evolution of the universe. Asseo and Sol (1987) emphasized the importance of Bianchi type-II universe. In present paper, we have investigated Bianchi type-II string cosmological models with time dependent bulk viscosity coefficient. The paper is organized as follows : The metric and field equations are presented in Section 2. In Section 3, we deal with exact solutions of the field equations with cloud of strings. In Section 4, we describe some physical and geometrical properties of the model in the presence and absence of bulk viscosity. Finally conclusions are given in Section 5.

2. THE METRIC AND FIELD EQUATIONS

We consider the line element for an anisotropic and homogeneous Bianchi type-II metric, given by

$$ds^2 = -dt^2 + A^2\left(dx^2 + dz^2\right) + B^2\left(dy - xdz\right)^2, \qquad [1]$$

where the metric potentials A and B are functions of t only. The energy-momentum tensor for bulk viscous string dust is given by Letelier (1983) and Landau & Lifshitz (1963) as

$$T_i^j = \rho v_i v^j - \lambda x_i x^j - \xi v^l_{;l}\left(g_i^j + v_i v^j\right), \qquad [2]$$

where v_i and x_i satisfy the condition

$$v_i v^i = -x_i x^i = -1, \qquad v_i x^i = 0, \tag{3}$$

ρ is the proper energy density for a cloud of string with particles attached to them, λ is the string tension density, v^i is the four velocity of the particles, and x^i is a unit space-like vector representing the direction of string. If the particle density of the configuration is denoted by ρ_p, then we have

$$\rho = \rho_p + \lambda . \tag{4}$$

Einstein's field equations (in natural units $8\pi G = 1, c = 1$) are given by

$$R_i^j - \frac{1}{2} R\, g_i^j = -T_i^j, \tag{5}$$

where R_i^j is the Ricci tensor, $R = g^{ij} R_{ij}$ is the Ricci scalar. In a co-moving co-ordinate system, we have

$$v^i = (0, 0, 0, 1), \qquad x^i = \left(A^{-1}, 0, 0, 0\right). \tag{6}$$

The Einstein's field equation (5) for Bianchi type-II space-time (1) with energy- momentum tensor (2) lead to the following system of equations :

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{B^2}{4A^4} = \lambda + \xi\theta, \tag{7}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3}{4}\frac{B^2}{A^4} = \xi\theta, \tag{8}$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} - \frac{B^2}{4A^4} = \rho, \tag{9}$$

where $\theta = v^l_{;l} = \left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right).$ [10]

Here, and in what follows, an over dot indicates ordinary differentiation with respect to *t*.

In order to obtain a more general solution, we assume (Wang 2005)

$$\xi = k\theta^m, \tag{11}$$

where k and m are the positive constants.

3. SOLUTIONS OF THE FIELD EQUATIONS

Equations (7)– (9) and (11) are four independent equations in five unknown parameters A, B, λ, ρ and ξ. One additional constraint relating these parameters is required to obtain explicit solutions of the system. We assume that the shear scalar (σ) is proportional to the expansion scalar (θ), which leads to

$$A = aB^n, \qquad [12]$$

where a is an integrating constant and n is a constant.

Substituting (12) into (10) and by using (11), we get

$$\theta = (2n+1)\frac{\dot{B}}{B}, \qquad [13]$$

$$\xi\theta = K\frac{\dot{B}^{m+1}}{B^{m+1}}, \qquad [14]$$

where $K = k(2n+1)^{m+1}$.

To obtain the determinate model of the universe, we assume that the sum of rest energy density and string tension density for cloud of string vanish (Reddy 2003, 2006; Mohanty 2007) i.e.

$$\rho + \lambda = 0. \qquad [15]$$

From equations (7), (9) and (15), we obtain

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 3\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = \xi\theta. \qquad [16]$$

By using equations (12) and (14), equation (16) leads to

$$\frac{\ddot{B}}{B} + 2n\frac{\dot{B}^2}{B^2} = \frac{K}{(n+1)}\frac{\dot{B}^{m+1}}{B^{m+1}}. \qquad [17]$$

Let $\dot{B} = f(B)$ which implies that $\ddot{B} = f\frac{df}{dB}$.

Hence equation (17) takes the form

$$\frac{df}{dB} + 2n\frac{f}{B} = \frac{K}{(n+1)}\frac{f^m}{B^m}. \qquad [18]$$

Equation (18) can be rewritten as (for m ≠ 1)

$$\frac{d}{dB}\left(f^{1-m}\, B^{2n(1-m)}\right) = \frac{(1-m)K}{(n+1)} B^{2n(1-m)-m} \qquad [19]$$

On Integrating (19), we obtain

$$f = \frac{dB}{dt} = \left[\frac{KB^{(1-m)}}{(2n^2+3n+1)} + LB^{2n(m-1)}\right]^{\frac{1}{(1-m)}}, \qquad [20]$$

where L is an integrating constant.

With the help of equations (12) and (20), the line-element (1) reduces to

$$ds^2 = -\left[\frac{KB^{(1-m)}}{(2n^2+3n+1)} + LB^{2n(m-1)}\right]^{\frac{-2}{(1-m)}} dB^2 + a^2B^{2n}\left(dx^2+dz^2\right) + B^2(dy - xdz)^2. \qquad [21]$$

Under suitable transformation of coordinates, equation (21) takes the form

$$ds^2 = -\left[\frac{KT^{(1-m)}}{\left(2n^2+3n+1\right)} + LT^{2n(m-1)}\right]^{\frac{-2}{(1-m)}} dT^2 + a^2T^{2n}\left(dx^2+dz^2\right)+T^2\left(dy-xdz\right)^2. \qquad [22]$$

In the absence of bulk viscosity K = 0, the metric (22) reduces to

$$ds^2 = -L^{\frac{-2}{(1-m)}}\ T^{4n}\ dT^2 + a^2T^{2n}\ \left(dx^2+dz^2\right)+T^2\ \left(dy-xdz\right)^2. \qquad [23]$$

4. SOME PHYSICAL AND GEOMETRICAL FEATURES

The rest energy density (ρ), the string tension density (λ), the particle density ρ_p, the scalar of expansion (θ) and the shear scalar (σ) for the model (22) are given by

$$\rho = -\lambda = n(n+2)\left[\frac{K}{(2n^2+3n+1)} + LT^{(2n+1)(m-1)}\right]^{\frac{2}{(1-m)}} - \frac{1}{4a^4T^{2(2n-1)}} \qquad [24]$$

$$\rho_p = 2n(n+2)\left[\frac{K}{(2n^2+3n+1)} + LT^{(2n+1)(m-1)}\right]^{\frac{2}{(1-m)}} - \frac{1}{2a^4T^{2(2n-1)}}, \qquad [25]$$

$$\theta = (2n+1)\left[\frac{K}{(2n^2+3n+1)} + LT^{(2n+1)(m-1)}\right]^{\frac{1}{(1-m)}}, \qquad [26]$$

$$\sigma = \frac{(n-1)}{\sqrt{3}}\left[\frac{K}{(2n^2+3n+1)} + LT^{(2n+1)(m-1)}\right]^{\frac{1}{(1-m)}}, \qquad [27]$$

$$\xi = \frac{K}{(2n+1)}\left[\frac{K}{(2n^2+3n+1)} + LT^{(2n+1)(m-1)}\right]^{\frac{m}{(1-m)}}, \qquad [28]$$

From equation (24) and (25), it is observed that the energy conditions $\rho \geq 0$ and $\rho_p \geq 0$ are satisfied in the presence of bulk viscosity for the model (22) when

$$T^{2(2n-1)}\left[\frac{K}{(2n^2+3n+1)} + LT^{(2n+1)(m-1)}\right]^{\frac{2}{(1-m)}} \geq \frac{1}{4n(n+2)a^4}. \qquad [29]$$

Under the realistic condition (29), we observed that

$$\lambda \leq 0, \qquad [30]$$

which shows that the string tension density is always negative. It is pointed out by Letelier (1983) that λ may be positive or negative. When λ < 0, the string phase of the universe disappears i.e. we have an anisotropic fluid of particles. It is seen that in the case $m<1$, the expansion scalar θ tends to infinitely large and the energy density $\rho \to \infty$ when $T \to 0$, but both θ and ρ tend to a finite value when $T \to \infty$ due to the presence of bulk viscosity. Hence the model represents on expanding universe with the big-bang start. However, in the case $m > 1$, it is observed that $\theta \to 0$ when $T \to \infty$, but θ tends to a finite value when $T \to 0$ due to the presence of bulk viscosity. Also, $\rho \to \infty$ when $T \to 0$ and $\rho \to 0$ when $T \to \infty$. Therefore the model describes an expanding universe without the big-bang start. From the above discussion, it is seen that the bulk viscosity plays a significant role in the evolution of universe. Further more, since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$, provided $n \neq 1$, the model does not approach isotropy at any time.

In the absence of bulk viscosity i.e. when $K = 0$, the physical and kinematical quantities for the model (23) are given by

$$\rho = -\lambda = \frac{n(n+2)b}{T^{2(2n+1)}} - \frac{1}{4a^4 T^{2(2n-1)}}, \qquad [31]$$

$$\rho_p = \frac{2n(n+2)b}{T^{2(2n+1)}} - \frac{1}{2a^4 T^{2(2n-1)}}, \qquad [32]$$

$$\theta = \frac{(2n+1)b}{T^{(2n+1)}}, \qquad [33]$$

$$\sigma = \frac{(n-1)b}{\sqrt{3}\, T^{(2n+1)}}, \qquad [34]$$

where $b = L^{\frac{1}{(1-m)}}$.

In the absence of bulk viscosity, the energy conditions $\rho \geq 0$ and $\rho_p \geq 0$ are satisfied for the model (23) when

$$T^4 \leq 4a^4 bn(n+2). \qquad [35]$$

The model (23) in the absence of bulk viscosity starts with a big-bang at $T = 0$ and the expansion in the model decreases as time increases. Since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$, provided $n \neq 1$, the model does not approach isotropy at any time. There is a real physical singularity in the model (23) at $T = 0$.

In the special case $m = 1$, equation (18) takes the form

$$\frac{df}{dB} + \alpha \frac{f}{B} = 0, \qquad [36]$$

where $\alpha = \frac{2n^2 + 2n - K}{n+1}$.

On Integrating (36), we obtain

$$f = \frac{dB}{dt} = MB^{-\alpha}, \quad [37]$$

where M is an integrating constant. In the same way as performed above, we can easily obtain

$$ds^2 = -M^{-2} T^{2\alpha} dT^2 + a^2 T^{2n}\left(dx^2 + dz^2\right) + T^2 (dy - xdz)^2. \quad [38]$$

$$\rho = -\lambda = \frac{n(n+2)M^2}{T^{2(\alpha+1)}} - \frac{1}{4a^4 T^{2(2n-1)}}, \quad [39]$$

$$\rho_p = \frac{2n(n+2)M^2}{T^{2(\alpha+1)}} - \frac{1}{2a^4 T^{2(2n-1)}}, \quad [40]$$

$$\theta = \frac{(2n+1)M}{T^{(\alpha+1)}}, \quad [41]$$

$$\sigma = \frac{(n-1)M}{\sqrt{3}T^{(\alpha+1)}}, \quad [42]$$

$$\xi = \frac{K M^m}{(2n+1)T^{m(\alpha+1)}}. \quad [43]$$

The energy conditions $\rho \geq 0$ and $\rho_p \geq 0$ are satisfied when

$$T^{2(2n-\alpha-2)} \geq \frac{1}{4n(n+2)a^4 M^2}. \quad [44]$$

It is observed that the expansion scalar θ is infinitely large when $T \to 0$ and θ tends to zero when $T \to \infty$ provided $\alpha + 1 > 0$. The energy density $\rho \to \infty$ when $T \to 0$ and $\rho \to 0$ when $T \to \infty$. Hence the model represents an expanding universe with a big-bang start. At the initial epoch i.e. at $T = 0$, the coefficient of bulk viscosity ξ is extremely large and it tends to zero when $T \to \infty$. Furthermore, since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$, provided $n \neq 1$, the model does not approach isotropy at any time.

5. CONCLUSIONS

In present work, we have studied Bianchi type-II string cosmological models based on exact solutions of Einstein's field equation with time dependent bulk viscosity in general theory of relativity. We have investigated anisotropic Bianchi-II space-time with and without bulk viscosity. To find a more general model we assume that the coefficient of bulk viscosity ξ is the power function of the expansion scalar θ i.e. $\xi = k\theta^m$. It is found that the power index m has significant influence on the models of the universe. There is a big-bang start in the model when $m \leq 1$ but there is no big-bang start when $m > 1$. In the models given by $m \leq 1$, the coefficient of bulk viscosity ξ is extremely large at initial epoch and tends to zero at late time (i.e. at present epoch), while in the model given by $m > 1$, the coefficient of bulk viscosity starts off with a finite value and tends to zero at present epoch. In each of our model the string tension density λ always appeared with negative value. Hence we would like to say that the string phases switched off or disappeared. The shear σ in the models behave in a similar way as that of expansion scalar θ and the models start with highest shear and end with shearless. Also, the ratio $\frac{\sigma}{\theta} \neq 0$, when $T \to \infty$ shows that our models never approach isotropy at any time. Our models are realistic and new to the others.

REFERENCES

Asseo E, Sol H (1987) Extra galactic magnetic fields. Physics Reports 148(6) : 307-436.

Avchar GR, Golechha DR and Tade SD (2021) Axially symmetric bulk viscous cosmological model in f (R, T) theory of gravity. J. Dynam. Syst. and Geom. Theor. 19(1) : 35-49.

Everett AE (1981) Cosmic strings in unified gauge theories. Physical Review D 24(1) : 858-868.

Kibble TWB (1976) Topology of cosmic domains and strings. J. Physics A : Mathematical and General 9 (8) : 1387-1398.

Kibble TWB (1980) Some implications of a cosmological phase transition. Physics Reports 67(1) : 183-199.

Kolb EW, Turner MS (1990) The Early Universe. Addison-Wesley Pub. Comp.

Krori KD, Chaudhuri T, Mahanta CR, Mazumdar A (1990) Some exact solutions in string cosmology. Gen. Relativ. Grav. 22(2): 123-130.

Landau LD, Lifshitz EM (1963) Fluid Mechanics 6, p. 505, USSR Acad. Sci. Moscow.

Letelier PS (1979) Clouds of Strings in general relativity. Physical Review D 20 (6) : 1294-1302.

Letelier PS (1983) String cosmologies. Physical Review D 28(10) : 2414-2419.

Mishra RK, Dua H (2019) Bulk viscous string cosmological models in Saez-Ballester theory of gravity. Astrophys. Space Sci. 364 (11) : 1-12.

Mohanty G., Sahoo RR, Mahanta KL (2007) Five dimensional LRS Bianchi type-I string cosmological model in Saez-Ballester theory. Astrophys. Space Sci. 312 (3-4) : 321-324.

Padmanabham T, Chitre SM (1987) Viscous Universe. Phys. Lett. A 120 (9) : 433-436.

Patel LK, Maharaj SD, Leach PGL (1996) Cosmic Strings in Bianchi II, VIII and IX space-times : Integrable cases. Pramana 46 : 331-339.

Rao VUM, Prasanthi UYD (2017) Bianchi type-II oscillatory string cosmological model in f (R, T) gravity. Prespacetime J. 8(4) : 451-458.

Reddy DRK (2003) A string cosmological model in Brans-Dicke theory of gravitation. Astrophys. Space Sci. 286(3) : 365-371.

Reddy DRK (2006) On string-Rosen cosmic strings in a scalar tensor theory of gravitation. Astrophys. Space Sci. 305 : 139-141.

Roy SR, Banerjee SK (1995) Bianchi type-II string cosmological models in general relativity. Classical and Quantum Gravity 11(8) : 1943-1048.

Sharma NK, Singh TK (2014) Bianchi type-II string cosmological model with magnetic field in f (R, T) gravity. Int. J. Theor. Phys. 53 (9) : 2912-2922.

Stachel J (1980) Thickening the string I. The string perfect dust. Physical Review D 21(8) : 2171-2181.

Vilenkin A (1981) Cosmi strings. Physical Review D 24(28) : 2082-2089.

Vilenkin A (1985) Cosmic strings and domain walls. Physics Reports 121(5) : 265-315.

Wang XX (2003) Exact solutions for string cosmology. Chin. Phys. Lett. 20 : 615-617.

Wang XX (2004) Bianchi type-III string cosmological models with bulk viscosity in general relativity. Chin. Phys. Lett. 22(1) : 29-32.

Weinberg S (1972) Gravitation and Cosmology : Principles and Applications of the general theory of relativity (New York : Wiley and Sons).

Yadav L, Yadav VK, Singh T (2012) Anisotropic Bianchi type-II massive string cosmological models with time-Decaying Λ term and thermodynamic parameters. Int. J. Theor. Phys. 51(10) : 3113-3126.

Zel'dovich Ya.B, Kobzarev IYu, Okun LB (1975) Cosmological consequences of a spontaneous breakdown of a discrete symmetry. Soviet Physics-JEPT 40(1) : 1-5.

Zel'dovich Ya.B (1980) Cosmological fluctuations produced near a singularity. Monthly Notices of the Royal Astronomical Society 192(9) : 663-667.